Types of Knowledge in Resolution of Problems with Double Integrals, using Mathematic Software*

Tipos de conocimiento en la resolución de problemas con integrales dobles, utilizando software matemático *

Tipos de Conhecimento em Resolução de Problemas com Integrais Duplos, utilizando Software Matemático

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Abstract

The applied mathematics are the Science who let the development that engineering has reached; however, solving problems had been the headache for students, due to the lack of virtual environments that let the development of concepts, skills, and procedures. The project aims to describe the application of different types of knowledge in problems resolutions of doubles integrals in students who make use of mathematic software in comparison to the traditional teaching. The labor had as a reference the theory of two stadiums translation and solution to solve problems according to Mayer. The investigation was quasi-experimental type. Is highlighted in the results difficulties in the translation phase because it showed a lower level toward the lecture comprehension, specifically of mathematic language in the students; although it evidenced that the 18% of students who approved the exam "pretest", while the 69% approved the exam "posttest"; It concludes about the importance of use of the mathematic software in the learning of vectoral calculus towards the problems resolution with double integrals; although, it reflected a positive change of the student by developing the activities with the use of the software.

Key Words: Superior teaching, Teaching of the mathematics, Assisted teaching by computer, Problem resolution, Educational technology.

Resumen

Las matemáticas aplicadas son las ciencias que permiten el desarrollo que la ingeniería ha alcanzado; sin embargo, la solución de problemas ha sido el dolor de cabeza para los estudiantes, debido a la falta de entornos virtuales que permiten el desarrollo de conceptos, habilidades y procedimientos. El proyecto tiene como objetivo describir la aplicación de diferentes tipos de conocimiento en resoluciones de problemas de integrales de dobles en estudiantes que hacen uso de software matemático en comparación con la enseñanza tradicional. El trabajo tuvo como referencia la teoría de la traducción de dos estadios y la solución para resolver problemas según Mayer. La investigación fue de tipo cuasiexperimental. Se destaca en las dificultades de resultados en la fase de traducción porque mostró un nivel más bajo hacia la comprensión de la clase, específicamente del lenguaje matemático en los estudiantes; aunque se evidenció que el 18% de los estudiantes que aprobaron el examen "pretest", mientras que el 69% aprobó el examen "posttest"; Concluye sobre la importancia del uso del software matemático en el aprendizaje de cálculo vectorial para la resolución de problemas con integrales dobles; aunque, reflejó un cambio positivo del estudiante al desarrollar las actividades con el uso del software.

Palabras clave: enseñanza superior, enseñanza de las matemáticas, enseñanza asistida por computadora, resolución de problemas, tecnología educativa.

Abstrato

A matemática aplicada é a ciência que permite o desenvolvimento que a engenharia alcançou; no entanto, a resolução de problemas tem sido a dor de cabeça para os alunos, devido à falta de ambientes virtuais que permitem o desenvolvimento de conceitos, habilidades e procedimentos. O projeto visa descrever a aplicação de diferentes tipos de conhecimento em resoluções de problemas de integrais de duplas em alunos que fazem uso de software matemático em comparação ao ensino tradicional. O trabalho teve como referência a teoria da tradução e solução de dois estádios para resolução de problemas segundo Mayer. A investigação foi do tipo quase experimental. Destaca-se nas dificuldades dos resultados na fase de tradução, pois apresentou um nível inferior para a compreensão das aulas teóricas, especificamente da linguagem matemática nos alunos; embora tenha evidenciado que os 18% de alunos que aprovaram o exame "pré-teste", enquanto os 69% aprovaram o exame "pós-teste"; Conclui sobre a importância do uso do software matemático na aprendizagem do cálculo vetorial para a resolução de problemas com integrais duplos; embora, refletisse uma mudança positiva do estudante desenvolvendo as atividades com o uso do software.

Palavras-chave: Ensino Superior, Ensino da Matemática, Ensino Assistido por Computador, Resolução de Problemas, Tecnologia Educacional.

Introduction

The applied mathematics had let the development that engineering has reached, even though during their professional training they are perhaps the biggest headache caused to the student (Hernández, 2016). The mathematic problem resolution, from the historical point of view has advanced in complexity from antiquity until actual science; therefore, should be assimilated new challenges, what immediately entails to the use of specialized mathematic software. (Toro-Carvajal, Ortíz-Álvarez, Jimenez-García, & Agudelo-Calle, 2012). The traditional education undergraduate level doesn't motive to the students to create their own strategies and techniques to solve problems (Rasmussen & Blumenfeld, 2007), this conceptual process isn't so attractive for the engineer students when they must study problems in a rigorously way in the classroom or in educative environments based on websites (Buitrago-Pulido, 2015).

For (Morrison, 2012), many students not only encounter great difficulty in demonstrating mathematical comprehension and strategic knowledge using the curriculum-based problem-solving technique, but students also struggle with the explanation of these mathematical processes in written form. Yet problem-solving skill is not what it seems. Indeed, the field of problem solving has recently undergone a surge in research interest and insight, but many ofthe results of this research are both counterintuitive and contrary to many widely held views. Pólya.' He discussed a range of general problem-solving strategies, such as encouraging mathematics students to think of a related problem and then solve the current problem by analogy, or to think of a simpler problem and then extrapolate to the current problem. It is possible to teach learners to use general strategies such as those suggested by Pólya, but that is insufficient. There is no body of ^research based on randomized, controlled experiments indicating that such teaching leads to better problem solving (Sweller, Clark, & Kirschner, 2010).

The National Council of Teachers of Mathematics (NCTM), the Iberioamerican Association of Institutions of Teaching of the Engineering (IBAITE), the Colombian Association of Engineer Faculties (CAEF) and the Colombia Nacional of Education Ministry (CNEM), agree that the mathematic for the engineer are a set of problems or situations whose treatment requires concepts, procedures and representations of different types who are intimately related. Being necessary the development of critical, reflective, and analytic thinking toward the problem resolution (Ortega, Duarte, & Lozano, 2016). This situation is no happening in classrooms specially in the engineering basic cycle. This reiterates it (Camarena, 2006), who affirm that mathematic teachers assume that problem resolution only is up to teachers of own engineering courses.

In the 21st century, however, the success of engineers and firms will be measured against how well they can adapt to new conditions and technologies. Thus to remain competitive in this global and knowledge-based economy and to ensure that the quality of life improves for everyone around the world, engineers must be educated differently. Our educators must instill within their students the belief that engineers are engaged in creative, stimulating, challenging and satisfying work that significantly improves the lives of people the world over. (G & Galloway, 2007). Solving problems is a persistent theme running through texts addressing what engineers do. Most all of the texts I analyze are the product of, or intended for, object-world reading and application. Engineering faculty see object-world knowledge as the hard cor oef engineering knowledge. It is knowledge of a powerful sort—the kind that can solve problems (Louis, 2009).

Engineering has strong connections to many other disciplines, particularly mathematics and science. Engineers use science and mathematics in their work, and scientists and mathematicians use the products of engineering—technology—in theirs. Engineers use mathematics to describe and analyze data and, as noted, to develop models for evaluating design solutions. Engineers must also be knowledgeable about science—typically physics, biology, or chemistry—that is relevant to the problem they are engaged in solving (Committee on Standards for K-12 Engineering & National Research, 2010).

The engineering always has been supported by mathematics; to most engineers vectoral calculus is one of the most important subjects that let a thinking formation to formulate, analyze and solve a wide range of problems of practice life. Like this, the authors (Eisenberg, 2002), (Tall, 1993), (Artigue & Ervynck, 1993), (Yudariah & Roselainy, 2004), (Willcox & Bounova, 2004), (Kashefi H. , Ismail, Yusof, & Rahman, 2011), coincide that for learning this subject it requires previous factoring knowledge, lineal equations system, derivative, integral and others. To (Schwarzenberger, 1980), "the function concept is a previous requirement to understand a lot of concepts. By not understanding it impacts in the learning of the following calculus, specially the functions in several variables" (p.159). These investigators are agreed that the difficulties in these pre-concepts will prevent a significative learning process in advanced mathematics even more when tedious calculus procedures have to be developed without the help of a mathematic software.

On the other hand, teachers in different educative systems and levels consider mathematics as extended and exhausted feeling under pression by the necessity of teach big quantities of contents in a very short time, due to the compromise with the students to get them ready for next level (Kashefi, Ismail, & Yusof, 2010). (Barb & Quinn, 1997), consider that the use of multiple methods to solve problems is advantageous, but it is also too risky due the time restriction. By their side (Rahman, Yusof, Ismail, Kashefi, & Firouzian, 2013), have developed investigations focused in the concept of functions of several variables with purpose of improving learning of the students through strengthening of problems resolution and mathematic thinking skills by technology tools that support the conceptual comprehension, allow them solve problems of their study field, showing some improvements but they consider that this effort is not enough yet.

For a long time, the mathematics education community has sought to embed the learning of mathematics in actual or concrete problems. The infusion of engineering-related ideas could be one way to accomplish that goal. However, the recently released common core state standards for mathematics do not even contain the word engineer or engineering (CCSSO & NGA, 2010). In contrast to science and technology standards docu-ments, which define technology in very broad terms, mathematics standards have tended to define technology more narrowly (i.e., as electronic tools) and do not refer to engineering at all, except as one of many fields in which mathematics is used (NCTM, 2000). Nevertheless, connections to engineering are implied in NCTM standards related to (1) problem solving and (2) making connections to subjects outside the mathematics curriculum.

The researchers (Cooper, Dann, & Pausch, 2000), argue is easy to say that the students "don't know how to solve problems"; but this affirmation is too simplistic, because they have reached certain level of mathematic competition and problem resolution at least through courses as Algebra and pre-calculus; what could be argued is that for many students have difficulties to learn how to write, proof and debug mathematic software where they need to learn why and how the program solves the problem. These technologic tools allow to provide an environment that students can learn the strategic types of problem resolution applying necessary concepts and skills that the teacher has taught in the mathematic teaching (Hernández, 2016).

As it is claim for (Mayer, 1986), an important objective of education is to help students to be effective problem solvers. People that can generate useful and original solutions when they face with problems never seen before. This author questioned: "Why mathematic problems are so hard to solve? Why is so hard to teach our children and teenagers how to solve these problems?", also (Skemp, 1987) claims that "Learning and teaching problems are psychologic and before we can advance in mathematic teaching, we need to know more about how to learn" (p.23). Initially is not recommended that the student solves to much problems, but with the solution of a few in a determinate time and with the use of technology tools, it could be understood each step of the process, building his own knowledge (Ortega, Lozano, & Tristancho, 2015).

The Engineering faculty students of Francisco of Paula Santander University (FPSU) that during their third semester they take the curse Vectoral Calculus and specifically the double integral subject, they are not outside to the problematic raised before. So, the elevated indexes of failed (64,2%) shown in the last year (Academic Information System FPSU, AIS). During the development of activities of classroom class and tutoring as a complement of independent work, difficulties are evident as: poor understanding of function concept, limits and continuity of a variable functions, their different forms of representation and of course, the problem resolution of application that involve these concepts. This is evident during the beginning and course of each academic period.

For above and to give sense and orientation to the investigation is formulated the following question: Are there differences between a group of Engineering students where is encourage the learing of double integrals based in problem resolution assited by mathematic software and those who do it in a traditional way? To try of answer the question and guide the investigation, it formuled the following goal: To Describe the application of different types of knowledge in problems resolution of double integrals, by students who make use of mathematic software in comparison with those who follow traditional classes.

Conceptual Framework

The problems. Any definition of problem should consist in three ideas: (1) the problem is currently in a state, but (2) you want this in another state and (3) there is not direct and obvious way to realize the change (Mayer, 1986).

Mayer Theory about mathematic problem resolution. To be able to give viability to the investigation, it has to start in what is need to know for a person to solve a mathematic problem. The work had as foundation the theory of two stages to mathematic problem resolution proposed by (Mayer, 1986), who raised that to solve with succesed a mathematic problem the resolver has to travel by two stadiums. In the translation stage it is require linguistic knowledge (knowledge about natural language words according to the situation), semantic (mathematic knowledge, signs, symbols and expressions inmersed in the problem) and esquematic (Identify and qualify problem typologies). While the solution stage requires operative knowledge (to know the procedures and calculations that are used in mathematics as algebraic and algoritmic process to get the problem answer) and strategic (By learning of tecniques (steps) and to know who to use diverse types of knowledge through problem resolution).

Cognitive Theory of multimedia learning. Multimedia means to show simultaneously words (spoken or written) and images (graphics, photography, animation, video). It is considered that it can be characterized multimedia material from three points of view: a) Technology (What devices are used? Monitor screen, projectors, speakers?), b) Presentation mode (verbal or pictorial), and c) Sensorial mode (visual or auditory). It rejects the analysis based exclusively in technology part and chooses the representation mode with some influence of sensorial mode. The authors (Mayer, R. E. & Moreno, R., 1998, Moreno, R. & Mayer, R. E., 1999, R. & Mayer, R. E., 2000 & Mayer, 2005) quoted by (Mayer R. , 2014), argue about the cognitive theoiry of multimedia learning it has its roots in dual codification theory by (Paivio, 2006). It refers to the reception process of new information in auditory and visual channels, to the procedure in short term memory and its subsequent integration with previous knowledge in long term memory.

Methodology

The investigation design was quasi-experimental type with test "pretest" and "postest" *ad hoc* made with an exhaustive scientific literature checking about the subject and adjusted to protocols to guarantees the validity and reliability of the results. The professor developed a traditional methodology, board use, workshops, support material, workshops in groups and ask for advice to the experimental group in both exams (pretest and postest). To answer the test pretest in both groups (control and experimental) only they had allowed to use: scientific calculator, pen, pencil, eraser and pencil sharpener. While the experimental group they received teaching based in a guided methodology to promote the mathematic thinking development to problems resolution as it set out for the knowledge theories porpoused by Richard E. Mayer; besides, the professor prepared the students in use of mathematic software (online) Wolfram Alpha, which allowed to be used in postest test.

These two groups, were evaluated through the problems resolution to find the volume of a superfice using double integral of Vectorial Calculus subject.

Those are the problems content in pretest exam:

Problem 1: The student must found the volume of solid that lies under the surface $x + 2y - z = 0$ and up of the region fenced by $y = x$ y $y = x^4$". Then should solve step by step the exercise.The student have to keep in mind that he must do a graphic of region D and later calculate the solid volume using the double integral.

Problem 2: You must found the volume circle by the paraboloid $z = x^2 + 3y^2$ and up of the fenced region by the planes $x = 0;\ y = 1; y = x; z = 0$". Solve step by step the exercise, keep in mind that you must do a graphic of region D and later calculate the solid volume using the double integral.
The two problems which cover postest exam are:

Problem 1: You must find the tetrahedron volume fenced by the coordinates of the planes and plane $x + y + z = 4$. Then solve step by step the exercise, keeping in mind that you must graphic the region D and later calculate the solid volume using double integral.

Problem 2: You must find the tetrahedron volume fenced by the coordinates of the planes and plane $z = 4 - 4x - 2y$. Then to solve step by step the exercise, to graphic the E region and D region and later to calculate the solid volume using double integral

Populaton and Sample

The population was constituted by a group of 80 students of Engeneering Facultie of Francisco of Paula Santander University, which coursed the subject Vectoral Calculus during I academic semester of 2016. The sample was not probalistic. The 60% of population is between 19 and 20 years old, 69% of students were in their third semester of their career, 90% of the control group study Mechanic Engeneering and 83% of experimental group are from System Engeneering.

Results

Keeping in mind the investigation questions, setted out objectives and the instrument aplication (pre test and post test test), for the steps stated bellow it is show the analysis of results. As a test of knowledge, the evaluation was based in the sum of the scores obtained by the student in each item. Were established two types of scales principaly, a dichotomic for whose answers that only admited to be evaluated as right or wrong: were established the 0 code for wrong answers and code 1 for right answers. About the process to plan, solve and express an answer, were evaluated with a graduaded scale with the following labels 0--> Wrong 1--> Insufficent 2--> Aceptable Solution y 3 -->Right solution. In this way, to more puntuaction better is the qualification of the student in the test.

Translate Stadium

Linguistic Aspects of pretest and postest exam.

Linguistic Knowledge of language is the meaning of words; he shows the model of question trhough of unique selection in both tests.

In the context of the problems that are dialed in the ***Vectoral Calculus*** mark the correct meaning of the following words:

1. Plane meaning:

a. It is something flat, smooth and without relieves
b. It is an element with two dimensions and contains infinities points and straights
c. It is a geographic representation of big terrain of land.
d. It is an element with three dimensions and contains infinities straights
e. None of them are correct.

The number 1 table it relates the pretest test items who consist the linguistic knowledge included by the two problems to calculate the surface volume with double integrals. Approximately 66,3% of group has the difficulty about the meanings of: *Tetrahedron, Fenced and Double integral;* while, around the 70% of students object of this study, they answered correctly about the meanings of *plane and particle*. Number II table for the same knowledge showed by posttest exam, the results show a 44% of the students group who didn't have clarity about the meaning of double integral, being the fundamental topic object of study. The same difficulty was showed during pretest exam.

Tabla 1.

Linguistic knowledge - Pretest

Pretest Exam	Incorrect		Correct	
	f	%	f	%
Meaning of Plane	36	45,0	44	55,0
Meaning of Particle	12	15,0	68	85,0
Meaning of Tetrahedon	47	58,8	33	41,3
Meaning of Fenced	51	63,8	29	36,3
Meaning of Double Integral	61	76,3	19	23,8

Source, Author

Tabla 2.

Linguistic Knowledge - Postest

Postest Exam	Incorrect		Correct	
	f	%	f	%
Meaning of Double Integral	65	81,3	15	18,8

Meaning of Plane	35	43,8	45	56,3
Meaning of Fenced	34	42,5	46	57,5
Meaning of Solid	15	18,8	65	81,3
Meaning of Tetrahedon	38	47,5	42	52,5

Source. Author

Semantic Aspects about Pretest and Posttest Exam.

The semantic knowledge refers to the facts about the world and the conceptual equivalences of the problem statements. Next, its shows one of the questions of semantic knowledge of pretest and posttest exam structured as an only selection question.

PART 2. In the context of the problems that are aborted in the Vector Calculation, mark the correct answer according to its equivalence:

$\iint_R f(x,y).dA$ It is equivalent to

a. $\underset{m \to \infty}{Lím} \sum\sum_{(i,j)} f(x_{ij}, y_{ij}) * \Delta x \Delta y$
b. $\underset{(m,n) \to \infty}{Lím} \sum\sum_{(i=1,j=1)} f(x_{ij}, y_{ij}) * \Delta x \Delta y$
c. $\underset{(m,n) \to \infty}{Lím} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}, y_{ij}) * \Delta x \Delta y$
d. $\underset{(m,n) \to \infty}{Lím} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}, y_{ij}) * \Delta A$
e. None of the above

The results of pretest exam are showed in table 3. It can be observed that approximately half of the group answered in a correct way with percentage that reach since 48,8% until 53,2% respectively. For the results of posttest test according to table 4, it can be observed that mostly of them have knowledge and competences to carry out conceptual equivalences with percentage between 65% and 82,5% respectively. 75% of the group has difficulties in the double integral equivalence. This is coherent with the same linguistic difficulty presented in both tests.

Tabla 3.

Semantic Knowledge - Pretest

Evaluative Items	Incorrect		Correct	
	f	%	f	%
Fenced Equivalence	37	46,8	42	53,2
Double Integral Equivalence	41	51,3	39	48,8
Region D Equivalence	28	35,4	51	64,6
Fubini Theoram Equivalence	41	51,3	39	48,8
Volume Equivalence	41	51,3	39	48,8

Source. Author

Tabla 4.

Semantic Knowledge Postest

Evaluative Item	Incorrect		Correct	
	f	%	f	%
Fenced Equivalence	25	31,3	55	68,8
Double Integral Equivalence	60	75,0	20	25,0
Region D Equivalence	14	17,5	66	82,5
Fubini Theoram Equivalence	36	45,0	44	55,0
Volume Equivalence	28	35,0	52	65,0

Source. Author

Schematic Aspects of Pretest and Posttest Exam.

The Schematic knowledge allows to identify the capacity of associate the problem statement with the topics thought in vectoral calculus. Table 5, shows a 55% of students with difficulties to associate the vectoral function subject with the statement; posttest results (see table 6 it improved significantly to 86.3% of students who answered correctly this knowledge, showing capacity of relate the problem statement with the subject.

Table 5. Schematic Knowledge –Pretest

Evaluative Item	Incorrect		Correct	
	f	%	f	%
Vectoral Function	44	55,0	36	45,0

Source. Author

Table 6. Schematic Knowledge – Posttest

Evaluative Item	Incorrect		Correct	
	f	%	f	%
Functions in Several Variable	69	86,3	11	13,8

Source. Author

Solution Stadium.

Operative Aspects of pretest and posttest exam.

The operative knowledge does reference to the mathematic thinking who has the student to carry a logic sequence of algebraic and algorithmic procedures to find the problem solution; being the factorization case the process with most difficulty when trying to solve the problems of application with double integral; another difficulty, is the application of a system of solution of equations (algebra) to find the dimensioning of D region in the cartesian plane *x* and *y*. The results of posttest test (see table 8), reach a 55% in each one of evaluated item with correct; however, between the evaluated results with acceptable, insufficient, and incorrect, they still evidence the same difficulties showed in pretest test.

Table 7.

Operative Knowledge - Pretest

Evaluative Item	Incorrect		Insufficient		Acceptable		Correct	
	f	%	f	%	f	%	F	%
Factorization Cases	27	37,5	18	25,0	8	11,1	19	26,4
Dimensioning Region D	27	37,0	15	20,5	7	9,6	24	32,9
Graphic Región D	22	31,9	18	26,1	12	17,4	17	24,6
Double Integral Definition	20	30,8	17	26,2	7	10,8	21	32,3
Double Integral Solution	14	21,9	24	37,5	9	14,1	17	26,6

Source, Author

Table 8.

Operative Knowledge - Posttest

Evaluative Item	Incorrect		Insufficient		Acceptable		Correct	
	f	%	F	%	f	%	F	%
Define Region D	4	5,1	15	19,2	12	15,4	47	60,3
Define Dimensioning	2	2,6	17	21,8	10	12,8	49	62,8
Graphic Region D	2	2,6	10	13,0	8	10,4	57	74,0
Define Double Integral	11	14,1	16	20,5	3	3,8	48	61,5
Solve Doble Integral	12	15,6	18	23,4	2	2,6	45	58,4

Source, Author

Strategic Aspects of Pretest and Posttest exam.

The percentage of table 9, identify the necessary steps and techniques trough problem resolutions in double integrals; the higher percentages are in the evaluative results of incorrect and insufficient; 61% of students showed difficulty in define the type of region dimensioned by the surface, being the strategic process fundamental to continue solving the problem correctly. This is an indicator which reflects the conceptual deficient bases that students possess trough the type of strategy or method to obtain the solution of a mathematic problem. Table 10 shows the results of posttest test, they are emphasized a 65% with correct, obtained by the group object of study in each one technique that are required to achieve the problems solution. The students had more acceptance and adaptation to an evaluative level trough teaching-learning process of types of knowledge according to Richard Meyer to resolve a mathematic problem, specifically, to find the volume of a surface using the double integral.

Table 9.

Strategic Knowledge – Pretest

Evaluative Items	Incorrect		Insufficient		Acceptable		Correct	
	f	%	f	%	f	%	f	%
Define D Region	25	42,4	11	18,6	10	16,9	13	22,0
Identify the Dimensioning	13	21,7	10	16,7	13	21,7	24	40,0
Development Values Table	19	32,8	4	6,9	10	17,2	25	43,1
Graphic D Region	12	20,3	12	20,3	11	18,6	24	40,7
Solve Double Integral	14	23,3	16	26,7	9	15,0	21	35,0

Source, Author

Table 10.

Strategic Knowledge - Posttest

Evaluative Items	*Incorrect*		Insufficient		Acceptable		Correct	
	f	*%*	F	%	f	%	f	%
Define D Region	*4*	*8,2*	7	14,3	2	4,1	36	73,5
Define Dimensioning	*4*	*8,2*	7	14,3	5	10,2	33	67,3
Values Table	*3*	*6,8*	7	15,9	2	4,5	32	72,7
Graphic D Region	*3*	*7,0*	6	14,0	5	11,6	29	67,4
Solve Double Integral	*2*	*4,7*	5	11,6	6	14,0	30	69,8

Source, Author

Relation of Total Scores of pretest and posttest exam.

To obtain the relation of total scores, it is taken the maximum punctuation that the students obtained in pretest and posttest exam equivalents to 76 and 83 points, respectively. It is made an equivalence to 5 points in 0 to 5 scales and establishing as a cut-off point the minimum value to the approbation, in this way 3 points are equivalent to 46 in pretest and 50 in posttest to obtain a relation of approved. The totality of students (82%) almost went out falling in pretest exam by the lack of dominance of preconceptions. This gap is diminished significantly in a 31% in total scores obtained in the posttest (see table 11); this result is due to the influence of improvement in the students of experimental group through the use of Wolfram Alpha software.

Table 11.

Tests' total results

Engineering Students		f	%
Score pretest	Failed	65	82%
	Approved	15	18%
	Total	80	100%
Score posttest	Failed	25	31%
	Approved	55	69%
	Total	80	100%

Source, Author

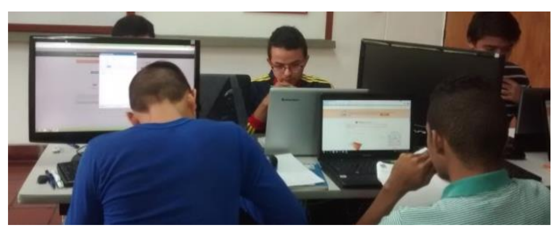

Figure 1. Use of the Wolfram Alpha Software.
Source, Author

figure 1. Shows the students of experimental group solving problems of double integrals of vectoral calculus with mathematic software.

Analysis of variances of a factor to the total scores in function of the groups experimental and control.

The variances analysis of a factor (ANOVA: initials in Spanish), allows to evaluate the presence of significant differences in the media scores of both groups; therefore, you can ask yourself: Does exist significant differences in the half values of the scores between pretest and posttest exams? In affirmative case; which type of mathematic thinking knowledge formulated by Mayer (1986) evidence those differences? The table 19 shows the results of variances analysis of a factor. Both results allow to assert that there are significant differences, around the 5% in the average values between the pretest and post test exams. In different moments of measurements; this is demonstrated by the values of $p=0,035<0,05$ and $p=0,024<0,05$ that reject the hypothesis of equality of the half and allows to assert that there are differences in the half values of the groups; it means, differ of the academic performance between the experimental and control group.

Table 12. ANOVA of a factor

		Sum of squares	gl	Half quadratic	F	Sig.
Total Pretest	Control Group	1,711	1	1,711	,841	,035
	Experimental Group	141,799	78	1,818		
	Total	143,510	79			
Total Postest	Control Group	,435	1	,435	,630	,024
	Experimental Group	87,105	78	1,117		
	Total	87,540	79			

Source, Author

Discussion

According (Mayer, 1986), the linguistic knowledge does reference to the translation that is to know the meaning of natural language of each one of the words that appear in the problem; to other authors as (Camarena, 2006), to this cognitive process it is called mathematical modeling, highlighting the transit that has to do the translator of the natural language to mathematic language and it is there where are presented large difficulties that can originate misunderstandings, because a mathematic misunderstandings it is produce when the problem solver builds a mental model of the problematic situation that is in conflict with the information of the problem statement (Mayer, Lewis, & Hegarty, 1992)

To understand it is required more that only reproduce information, the student can not understand the statement of a problem, doesn't understand almost any mathematic concepts of writing or about what they write; it means that understand is the ability of think and act with flexibility from what you already know. This idea of comprehension view from the performance contrasts with other vision of preeminent understanding both in our everyday language as in cognitive science (Perkins, 1999). Through this cognitive process according with (Sabagh Sabbagh, 2008), the student must have capacity to do those algebraic and algorithmic process that are required to the solution of a problem; this means, to build the representation of a problem and to calculate its solution of two phases that are interacting but that they aren't using the same mental process, for the first step it must categorize and for the second one is the capacity of calculate (Descaves & Butlen, 1999).

According with (Leonard, Gerace, & Dufresne, 2002), it is believed that the value of a deep comprehension of the concepts is being capable of apply the knowledge flexibly to solve problems not familiars; the ability of problem resolution without conceptual comprehension is not valued by the majority of teachers; because of that the results of the investigation test are accord to the conclusions of (Rasmussen & Blumenfeld, 2007), which claim that the traditional education doesn't motivate to the students to create their own strategies and techniques to solve problems. Actually, there is the necessity to implement virtual environments with a new methodologic paradigm both for research as for the generation of new strategies and educative process (Ardila-Rodríguez, 2011); the same for (Ávila-Fajardo & Riascos-Erazo, 2011).

The inclusion of TIC (initials in Spanish) is a process that requires a constant evaluation; therefore, teachers must be familiar and updated permanently to technologic tools, so the investigators concluded (Guacaneme-Mahecha, Gómez-Zemeño, & Zambrano-Izquierdo, 2016). For the National Council of Teacher of Mathematics (2014), this preparation will allow the calculus teacher in their pedagogic practices to use the teaching-learning through the problem resolution improving in the student the instructional processes with: interaction, individual attention, experiences amplification of students and self-control of the comparative learning using technologies in an effective way keeping in mind that these tools are essential in order to obtain an education of high quality in mathematics (Goldsmith, Doerr, & Lewis, 2014).

Conclusions

Keeping in mind the results of the instrument analyzed, a big majority of students reached a positive labor to the linguistic knowledge with a good domain of the understood vocabulary by the two problems in each test. The operative knowledge is the cognitive process with major difficulty; specifically, in pre-concepts as the factorization cases, solution of equations system and clear a variable. The major difficulty in all the knowledges raised by Richard Mayer to the present investigation was the strategic knowledge, for the student is hard to express their own words and in mathematic terms the steps to follow to achieve the problem solution; this is because the students aren't used to being evaluated with this type of instrument

The 18% of students approved the pre-test test, however the 69% approved the post-test test; therefore it is concluded in general terms the importance of the use of mathematic software in the learning of calculus through the resolution of problems allowed to the students of experimental group (for the current investigation) to obtain a better performance in the post-test test; although it reflected during the instrument application a positive change of the student in activities developed with the software use.

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